

Table 2.1. The z transforms of commonly used sequences.

$x(n)$	$X(z)$	Convergence region
$\delta(n)$	1	$z \in \mathbb{C}$
$u(n)$	$\frac{z}{(z-1)}$	$ z > 1$
$(-a)^n u(n)$	$\frac{z}{(z+a)}$	$ z > a$
$nu(n)$	$\frac{z}{(z-1)^2}$	$ z > 1$
$n^2 u(n)$	$\frac{z(z+1)}{(z-1)^3}$	$ z > 1$
$e^{an} u(n)$	$\frac{z}{(z-e^a)}$	$ z > e^a$
$\cos(\omega n) u(n)$	$\frac{z[z - \cos(\omega)]}{z^2 - 2z \cos(\omega) + 1}$	$ z > 1$
$\sin(\omega n) u(n)$	$\frac{z \sin(\omega)}{z^2 - 2z \cos(\omega) + 1}$	$ z > 1$
$\frac{1}{n} u(n-1)$	$\ln\left(\frac{z}{z-1}\right)$	$ z > 1$
$\sin(\omega n + \theta) u(n)$	$\frac{z^2 \sin(\theta) + z \sin(\omega - \theta)}{z^2 - 2z \cos(\omega) + 1}$	$ z > 1$
$e^{an} \cos(\omega n) u(n)$	$\frac{z^2 - ze^a \cos(\omega)}{z^2 - 2ze^a \cos(\omega) + e^{2a}}$	$ z > e^a$
$e^{an} \sin(\omega n) u(n)$	$\frac{ze^a \sin(\omega)}{z^2 - 2ze^a \cos(\omega) + e^{2a}}$	$ z > e^a$

Therefore, for a linear system, given $X(z)$, the z -transform representation of the input, and the coefficients of its difference equation, we can use equation (2.120) to find $Y(z)$, the z transform of the output. Applying the inverse z -transform relation in equation (2.36), the output $y(n)$ can be computed for all n .¹

¹ One should note that, since equation (2.120) uses z transforms, which consist of summations for $-\infty < n < \infty$, then the system has to be describable by a difference equation for $-\infty < n < \infty$. This is the case only for initially relaxed systems, that is, systems that produce no output if the input is zero for $-\infty < n < \infty$. In our case, this does not restrict the applicability of equation (2.120), because we are only interested in linear systems, which, as seen in Chapter 1, must be initially relaxed.